



Speaker: Sergei Tabachnikov
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Friday, October 26, 2012
3:00 PM
127 Hayes-Healy Hall

Title: Tire tracks geometry, hatchet panimeter, Menzin's conjecture, and complete integrability

Abstract:

This talk concerns a simple model of bicycle motion: a bicycle is a segment of fixed length that can move in the plane so that the velocity of the rear end is always aligned with the segment. The trajectory of the front wheel and the initial position of the bicycle uniquely determine its motion and its terminal position; the monodromy map sending the initial position to the terminal one arises. This circle mapping is a Moebius transformation, a remarkable fact that has various geometrical and dynamical consequences. Moebius transformations belong to one of the three types: elliptic, parabolic and hyperbolic. I shall outline a proof of a 100 years old conjecture: if the front wheel track is an oval with area at least π then the respective monodromy is hyperbolic. I shall also discuss the related Backlund-Darboux transformation on curves, in the continuous and discrete settings, its complete integrability, and its unexpected relation with the binormal (smoke ring, filament) equation, a much studied completely integrable PDE.