Discrete Mathematics Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

Speaker: **Adam Hammett**
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Tuesday, October 9, 2012
1:00 pm
117 Hayes-Healy Hall

Title: How often are two permutations comparable?

Abstract:
Permutations p=p(1)p(2)...p(n) and q=q(1)q(2)...q(n) of [n]=1,2,...,n are comparable in Bruhat order if one can be obtained from the other through a sequence of exchanges of inverted elements (i.e. exchanging elements p(i) and p(j) if p(i)¿p(j) with i¡j). Permutations p and q are comparable in weak order if one can be obtained from the other through a sequence of exchanges of adjacent inverted elements. Both the Bruhat and weak orderings induce a partial ordering on the collection of permutations of [n]. To get a handle on how often two permutations will even be comparable in these orderings, we estimate the size of Prob[p ¡ q] for permutations p and q selected independently and uniformly at random for the two cases of "¡" meaning Bruhat or weak ordering. We arrive at the somewhat surprising conclusion that in either case, this almost never happens! Said formally, Prob[p ¡ q] tends toward 0 as n tends to infinity, and we present explicit rates of convergence to ! 0 in each case. This is joint work with Boris Pittel.