

**Archive of Colloquia Abstracts**  
**2012-2013**  
**Department of Mathematics**  
**University of Notre Dame**

**The Nash problem on families of arcs**

Tommaso de Fernex (University of Utah)

Date: September 7, 2012

Host: Nero Budur

Abstract:

Hironaka's fundamental theorem on resolution of singularities allows to study the geometry of any complex manifold with singularities by associating to it another complex manifold that is smooth everywhere and is "close enough" to the original one.

A more intrinsic approach to study singularities of complex manifolds was proposed by Nash. The idea is to look at the space of arcs (i.e. analytic germs of curves) passing through the singular points. This space decomposes into finitely many irreducible families, and carries much of the information encoded in a resolution.

The Nash problem gives a precise formulation of how such families of arcs should relate to resolutions of singularities. In this talk I will give an overview of the history and solution of the problem.

This will be a colloquium-level talk and will be accessible to a broad audience.

## Interpolation, arithmetic and Birch and Swinnerton-Dyer style conjectures

Andrei Jorza (California Institute of Technology)

Date: September 26, 2012

Host: Nero Budur

Abstract:

The values of the Riemann  $\zeta$  function at negative integers are rational numbers computable in terms of the Bernoulli numbers, and the  $\zeta$  function can be thought of as a meromorphic interpolation of the Bernoulli numbers. It is not a very strong interpolation since the negative integers are not dense in  $\mathbb{C}$ , but they are dense in the ring of  $p$ -adic integers. An arithmetic property of the Bernoulli numbers allows one to interpolate them into an *analytic* function  $\zeta_p$  defined not over  $\mathbb{C}$  but over the  $p$ -adic integers; such analytic functions exist in more general contexts.

The underlying principle of the Birch and Swinnerton-Dyer conjecture (one of the Clay problems) is that the first term in the Taylor expansion of an  $L$ -function (which is a generalization of the Riemann  $\zeta$ -function) contains rich information about arithmetic. We will discuss the Bernoulli numbers, the Riemann  $\zeta$ -function, the function  $\zeta_p$ , and analogs of the above mentioned principle for  $p$ -adic  $L$ -functions, which are analogs of  $\zeta_p$ . This field of research has become extremely active in the last decade.

## **Linearity, nonlinearity and medical imaging**

Athanasios Fokas (University of Cambridge)

Date: September 28, 2012

Host: Alex Himonas

Abstract:

A new method for analyzing boundary value problems (BVPs) for linear and integrable nonlinear PDEs, extending ideas of the inverse scattering transform method, was introduced in [1] and further developed by several researchers. First, this method will be introduced by using linear evolution PDEs in the half-line as illustrative examples. Then, for the integrable nonlinear analogues of these problems it will be emphasised that: (i) For linearizable BVPs the new method is as effective as the usual inverse scattering transform method; this includes PDEs with a third order derivative, for which the alternative approach of extension from the half-line to the full line, fails. (ii) For general (non-linearizable) BVPs, it yields the solution via a matrix Riemann- Hilbert problem, whose jump matrix can be characterized via a well defined nonlinear equation, in terms of the given initial and boundary conditions. (iii) For general BVPs, with either decaying, or  $t$ -periodic boundary conditions, it yields effective long time asymptotic formulae. Finally, the related development of the emergence of a new analytical approach to inverting integrals with applications in medical imaging, will be mentioned. [1] A S. Fokas, A Unified Transform Method for Solving Linear and Certain Nonlinear PDEs, Proc. R. Soc. Lond. A 453, 1411-1443 (1997).

## **Geometric aspects of Tensor Analysis**

Lica Chiantini (University of Siena - Italy)

Date: October 24, 2012

Host: Juan Migliore

Abstract:

Tensors, i.e. multivariate matrices, are a natural tool for the study of several fields of Mathematics. Despite their large applicability, many basic questions on the (multi-)linear Algebra of tensors are still unknown. Recently, results on secant varieties to projective varieties provided a new point of view and some advances in the theory of tensors. In the talk, I will illustrate an introduction to the connections between algebraic properties of spaces of tensors and the geometry of Segre varieties, with a view toward new open problems that the interaction suggests.