UNIVERSITY OF NOTRE DAME

SPECIAL SEMINAR

Department of Mathematics

Guest Speaker: Nick Freeman

University of Michigan, Ann Arbor

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Lecture Title:

Dynamics of the Secant Method and Other Monomial-Type Maps

Date: Friday, December 4, 2026

Time: 2:00 PM

Location: 258 Hurley Hall

Abstract:

We discuss the secant method, a root-finding algorithm alternative to Newton's method. Given a rational map $f : \mathbb{P}^1 \to \mathbb{P}^1$, the secant method yields a dynamical system S_f on $\mathbb{P}^1 \times \mathbb{P}^1$ (minus a finite indeterminacy locus). If $r \in \mathbb{C}$ is a simple root of f, then the point (r, r) is an attracting fixed point of S_f. Additionally, when 0 is a simple root of f with $f''(0)$ nonzero, we have that S_f is an instance of the broader class of ``monomial-type maps." A ``monomial map" is simply a function whose output coordinates are all monomials in the input coordinates. A ``monomial-type map" is then a coordinate-by-coordinate scaling of a monomial map by some holomorphic scalar functions. In the aforementioned case for S_f, we find that $S_f(x, y) = (a(x, y) * xy, x)$. By extending the 1-dimensional constructions of Böttcher coordinates and Green's functions to these higher-dimensional monomial-type maps, we can show that (1) S_f has well-defined dynamical potential, (2) it takes the v equipotential to the v^ϕ equipotential (where ϕ is the golden ratio), and (3) S_f is locally modeled by the Fibonacci monomial map $(x, y) \mapsto (xy, x)$, whose dynamics are well-understood.