

Course title: Instantonic field theories

Course description:

We will study a class of topological quantum field theories in which certain special correlation functions are computed by counting special field configurations — “instantons” — and thus yield answers to problems of enumerative geometry. The main examples will be:

(1) Witten’s supersymmetric quantum mechanics (counting gradient trajectories of a Morse function).

(2) A-model and Gromov—Witten invariants (counting holomorphic maps). An example of an application here is the answer (due to Kontsevich—Manin) to the question: how many rational curves of degree d pass through $3d-1$ points in general position on a (complex projective) plane?

(3) Donaldson—Witten theory (counting anti-self-dual connections on a 4-manifold). This example has crucial applications to the study of smooth structures on 4-manifolds.

In field theories of this type, the path integral can be recognized as the Mathai-Quillen representative for the Euler class of a certain (infinite-dimensional) vector bundle, equipped with a preferred section, whose zeros are the instantons.