MATH 80210: Topics in Algebra (Fall 2026)
Quantum Groups

Li, Yu

Course Description

This course is an introduction to the theory of quantized enveloping algebras associated with complex semisimple Lie algebras. Quantized enveloping algebras provide a rich deformation theory of classical Lie theory, connecting representation theory, noncommutative algebra, algebraic geometry, and Poisson geometry.

The course will begin with the construction and structure theory of quantized enveloping algebras, including their Hopf algebra structure, triangular decomposition, and PBW-type bases. We will develop the theory of finite-dimensional representations, including highest weight modules and quantum analogues of classical representation-theoretic constructions. We will then introduce crystal bases and their applications to the combinatorics of representations. Time permitting, we will explore the semiclassical viewpoint: quantum groups as quantizations of Poisson Lie groups and their Poisson homogeneous spaces.

The primary reference will be:

> J. C. Jantzen, *Lectures on Quantum Groups*, Graduate Studies in Mathematics, Vol. 6, American Mathematical Society.

Prerequisites

A standard graduate algebra background is expected, including familiarity with groups, rings, modules, tensor products, and basic representation theory. Prior knowledge of semisimple Lie algebras, root systems, and highest weight representations will be helpful, but will not be assumed. The necessary background from Lie theory will be developed as needed during the course.

Topics

1. Review of Lie-theoretic background
2. Construction of quantized enveloping algebras
3. Finite-dimensional representations
4. PBW bases and structural results
5. Crystal bases
6. Quantum groups and Poisson geometry

Additional Topics (depending on time and interest)

1. Cluster algebra structures on quantized enveloping algebras
2. Quantized enveloping algebras at roots of unity