Book review: The Mathematical Gazette

Calculus in context: background, basics, and applications, by Alexander J. Hahn, 712 pp, £70.50, ISBN 978-1-4214-2230-5 (hardback), Johns Hopkins University Press (2017).

Calculus is conceptually beautiful. It has a long and interesting history, together with applications in science, engineering and much else. There will therefore always be new books on calculus, even if there is little expectation of one with a refreshingly new approach to the subject itself. Unfortunately, in too many such texts, calculus is introduced without much context, and the applications are mostly trivial, routine or contrived. For a beginner trying to understand calculus, much of the difficulty lies in the rather unusual notation being adopted by practitioners. Thus, a good calculus book has to be reasonably comprehensive, particularly in the historic context, and it should include convincing examples to illustrate the theory, together with plenty of realistic applications and well chosen exercises for the reader.

For this massive text, the mathematical preliminaries of geometry, trigonometry, algebra, and coordinate geometry are developed in a leisurely pace in the first four of six chapters forming the first of two parts. Such topics are set out within the historic frame of the ancient Greeks and the heliocentric revolution in astronomy; thus the genius of Archimedes and the vital contributions from Copernicus, Tycho Brahe, Kepler and Galileo are discussed in full and in the proper context. The rather interesting problem of Kepler's wine barrel is rarely mentioned in textbooks. In November 1613, Kepler celebrated his second wedding, the occasion for which he had to supply the wine. Apparently the wine merchant determined the volume of the barrels without any regard for their shape and just inserted a rod diagonally through the central hole until its tip reached the far inner edge of the barrel, and the volume was then read off from the markings on the rod. As Kepler put it: *I was puzzled that this method could provide an accurate measure of the content. It seemed to me that, as a newlywed, it was incumbent that I should investigate this method with geometric principles and to bring to light any relevant laws.* His second marriage was thus off to a good start, and history would tell us that it was much happier than his first. Anyway, the notion of a function is introduced, and the last two chapters in the first part are on the calculus of Leibniz and of Newton.

A comprehensive treatment of calculus requires more depth and, in the second part, the differential and the integral calculus are covered in two separate chapters, each followed by its own chapter on applications. The vexed question, raised by Bishop Berkeley in 1734, of whether a method which delivers correct results necessarily means that the argument involved must be sound, is aired. It is then pointed out that 100 years later the likes of Cauchy, Riemann and Weierstrass reformulated the fundamental concepts and provided the calculus of functions with the foundation that gave validity of its assertions; in particular, the notion of a limit is defined, and well illustrated with examples. Thus, besides the pioneering work of the Bernoullis and Euler, important results involving the intermediate value, compactness and the fundamental theorem are stated clearly, with the emphasis that although the results are intuitively clear subtle analysis is required for the proofs.

When it comes to applications, many books often either just pay lip-service to, or give only the very rudimentary aspects of, the problems involved. Here, the author has taken on the task of laying down also the necessary background to the associated problems in a leisurely pace before giving the details in both the mathematics and the physics or engineering involved. After all, an equation and its solution need to make more than just mathematical sense and, in any case, the better understanding of the physical problem the more appreciation of the mathematics. The reader is treated to a variety of interesting, realistic and practical problems with rather thorough discussions. Besides the usual problems of applying the derivative to rates of change in the growth of organism, radioactive decay, or cost of production, there is a substantial piece on the building of a suspension bridge, giving the accompanying analysis associated with the Brooklyn Bridge over the East River, and the George Washington Bridge over the Hudson River, in New York. Also, experiments reputed to have been carried out by Galileo are given a fair airing, and there is a long section on the theory of the refracting and reflecting telescopes. In applications of the integral calculus, there is the problem of estimating the weight of domes, taking as examples two of the world's historic buildings: the Hagia Sophia and the Roman Pantheon. Action of forces in general, and calculations involved in the motion of a planet, and bullets in a gun barrel, in particular, are covered much more throughly than in most texts on calculus.

The next chapter is on the basics of differential equations, dealing only with first and second order equations, for which complex numbers and power series are introduced. In considering systems with springs and damping elements, besides returning to the problem of hanging cables associated with a suspension bridge, the theory is also illustrated with shock absorber experimental data from the motor industry. The last chapter is on polar calculus and Newton's planetary orbits, in which the full relationship between the conic section and the inverse square law is derived and explained. The final sections are on gravity and geometry, with illustrations from spiral galaxies.

Each chapter ends with a large set of problems and projects, varying from simple exercises to lengthy guides and pointers to investigations in wide-ranging topics. There are, for example, problems associated with the Global Positioning System and, if you are a bridge fan, there are data associated with bridges from all over the world, including our Humber Bridge. The text is adorned with countless useful diagrams, tables and relevant photographs.

The book can be described as an excellent one on general science and engineering, together with an appropriate amount of the required mathematics associated with the discoveries and inventions from human endeavours over the centuries. It is very well written in an engaging and enthusiastic style: it is very suitable for first year students, is perhaps not too demanding for students about to enter university, and it is particularly useful to those with more than a passing interest in astronomy. There is plenty to learn for the reader, and the massive text is also a good reference book on calculus. This labour of love from the author more than satisfies the high hopes for a good calculus book that I set out in the first paragraph of the review, and I highly recommend it.

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