Book Reviews

Mathematical Excursions to the World's Great Buildings

Alexander J. Hahn
Princeton University Press, 2012, ISBN 978-0-691-14520-4
Distributed in Australia by Footprint Books

This fascinating book recounts the history of monumental buildings in the Western tradition, interspersed with the mathematics pertinent to their design and construction.

The primary narrative focuses on the architectural form and structure of iconic buildings ranging from the Parthenon in Athens to the Sydney Opera House. The tone of the volume is set by the stunning cover, a photo of the Opera House at sunset with background a working drawing of elements of its design.

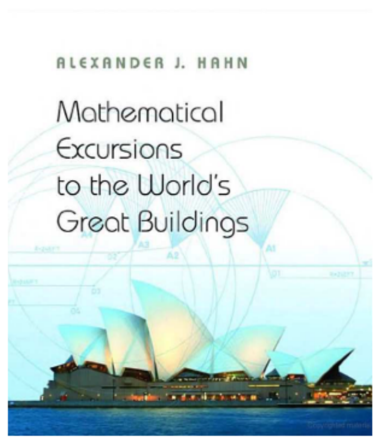
The secondary narrative concerns the mathematics behind the architecture, including plane and solid Euclidean geometry, analytic geometry, vector algebra and some basic calculus. These mathematical snippets are not systematically developed but scattered throughout the text in appropriate places determined by the architectural narrative. More advanced mathematics needed for the proper analysis of the structures, such as fluid mechanics, splines and finite element methods, are mentioned but not presented.

Greek architecture was based on aesthetics rather than structural analysis. The mathematical background is elementary geometry, including the Golden Mean and the construction of regular polygons, although the construction of the Acropolis buildings (-400) such as the Parthenon as well as the enormous conical theatre of Epidaurus (-360) preceded the formalisation of Euclid (-300). No doubt there was a deal of trial and error involved, as well as intuition and experience, but there remains no record of study of strength of materials or stress analysis.

The most important architectural advances of the Romans were undoubtedly the semi-circular arch and the dome, for example the Pantheon (120). Here an analysis of the stresses induced by the structure and the loads it carried would seem to be essential, and in fact the treatise of Vitruvius (-100) contains some of the theory needed. For example, Vitruvius describes the thickness of the piers necessary to counteract the horizontal component of the stress in an arch, as well as the use of high-density concrete in the lower parts of the structure and lower density in the higher. The success of Roman construction methods is attested by the massive aqueducts still standing in Italy and the colonies. Another innovation due to the Romans was the use of ovals rather than circles in stadiums and buildings, for example the Colosseum. The Roman oval was formed by four arcs of circles of two different radii rather than true ellipses, perhaps for ease of laying out. The relevant mathematics described here includes this geometric construction, as well as the decomposition of forces into components, and its inverse, the vector resultant of forces.

The construction of ever larger domes was a feature of ecclesiastical buildings in the Middle Ages, both Islamic and Christian. The book describes several notable examples, including the Hagia Sofia in Istanbul (535), a cathedral later converted to a mosque and the Great Mosque in Cordoba (800), a mosque later converted to a church. (The dates given in this review are only approximate, because construction often extended over hundreds of years, in many cases requiring extensive reconstructions due to damage by structural failure, fire or earthquakes.) The Hagia Sofia is one of the earliest examples of reinforced concrete construction, in the sense that circular chains were incorporated into the masonry during repairs, to balance the hoop stress which had cracked the dome. The mathematics in this section includes stress analysis in an arch and the symmetries involved in Islamic decoration.

The architectural innovations of the of the later Middle Ages are the splendid Gothic cathedrals characterised by ribbed vaults consisting of several crossing Gothic arches with a common vertex, flying buttresses and the large stained glass rose windows, a notable example being the Cathedral of Notre Dame in Chartres (1220). The mathematical section includes stress analysis in a pin-jointed truss, and the geometry of Gothic arches.



The flower of the Renaissance is the Cathedral of Florence, whose ribbed vault dome was designed by Brunelleschi (1377–1446). This 45 m-wide dome is supported by the structural strength of the Gothic arches themselves, without relying on buttresses. Another important example is the Duomo of Milan (1420), because we have extensive records of the Building Council which oversaw the design and construction. Others are St Mark's Basilica (1063) and the Doge's Palace (1309) in Venice, and the Cathedral and the Leaning Tower in Pisa (1300). The mathematical examples in this section include true ellipses, the geometry of the regular polytopes and decimal arithmetic. On the applied side, we have the study of strength of materials and stress in beams, arches and domes including an analysis of the stress distribution in the dome of the Florence Cathedral.

The High Renaissance saw Palladio's (1508–1580) villas and churches, and St Peter's Cathedral in Rome (1500), due to Bramante (1444–1515) and Michaelangelo (1475–1564), the oval Colonnade of St Peter's Square being designed by Bernini (1598–1670). The mathematical discussion in this section is devoted to perspective and conic sections.

Architecture in the Age of Reason is typified by St Paul's Cathedral in London. Its architect Wren (1632–1723) was familiar with the dictum of Hooke (1635–1703): 'As hangs the flexible line, so but inverted will stand the rigid arch'. Wren's design of St Paul's has a triple dome: the inner and outer dome are decorative hemispheres, but the load-bearing intermediate dome approximates a surface of revolution based on an inverted catenary. This method was later copied in the domes of St Isaac's Cathedral in St Petersburg, as well as the Capitol in Washington. Other innovations of this era include the use of cast and wrought iron in construction.

The design and construction of the Sydney Opera House occurred at a crucial time (1957–1973) in the history of architecture. Pre-stressed concrete was widely accepted; computers were enabling massive computations; software for finite element methods was developed. On the other hand, computer-aided design and manufacture (CAD–CAM) was not yet available, so Utzon's imaginative free-form roof design proved impossible to analyse mathematically. After experimenting with various geometries, Utzon, in conjunction with structural engineer Ove Arup, eventually settled on a design based on spherical triangles cut from a sphere of radius 75 m. As well as finite element analysis, scale models of the roof were tested in wind tunnels. The innovations in the design were not limited to the sail structure of the roof, but also extended to the podium. This was a concourse of area 116 m by 186 m largely unimpeded by columns. The roof was supported by pre-stressed concrete beams of length 75 m and width 2 m whose cross-section varied continuously from U-shaped at the supports, where shear stress is maximal, to T-shaped at the centre, where bending stress is maximal. The mathematics in this chapter is mainly concerned with spherical geometry but also includes the analysis of a hanging chain, both free-hanging and loaded, and the stress distribution in its inverted version, like Saariinen's Gateway Arch in St Louis (1965).

The book contains a final chapter wholly concerned with calculus and its application to building design and construction. Because of its lack of an underlying theory of limits, it should be regarded mainly as a review addressed to an audience who have already studied calculus and are interested in its applications.

To summarise, this intriguing book whose author, Alexander Hahn, is a professor of mathematics at the University of Notre Dame, Indiana, should be of interest to mathematicians who want to understand how their subject has been applied to produce some of the marvelous monuments of the world, and to amateurs of the history of architecture who wish to gain a deeper understanding of the mathematics behind some of those buildings.

Phill Schultz
School of Mathematics and Statistics, The University of Western Australia, Crawley, WA 6009, Australia. Email: phill.schultz@uwa.edu.au

◇ ◇ ◇ ◇ ◇ ◇