

Part Two, called 'Time and Space', contains thoughts on mathematical motion. It starts with the introduction of two basic concepts: coordinate systems and dimension. Using them, motion can be described as a numerical relationship (a function). Leading the reader step by step through several problems, Paul Lockhart introduces a vector representation of motion and then moves to mechanical relativity. It is inspiring to read this ingenious approach to the differential calculus and its numerous applications. The final words of the book encourage the reader to explore mathematics, which the author compares to a jungle – one may travel around it, following one river or another.

This attractive book is written for a wide range of readership. Whether or not the reader has a sound mathematical background, they will find delight in exploring the mathematics introduced in these pages. Students of upper secondary and higher levels, teachers, mathematicians, scientists, and people looking for mathematical entertainment are all invited in this book to explore the world of mathematics. It is a piece of art, a poem about the excitement and pleasure of doing mathematics.

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Mathematical excursions to the world's great buildings, by Alexander J. Hahn. Pp. 318. £34.95 (Hardback). 2012. ISBN: 978-0-691-14520-4 (Princeton University Press).

This is a beautifully produced book, describing the mathematics behind some of the world's most important and spectacular buildings. Mathematics and architecture both involve shape, pattern and structure, so it is not surprising that they relate to one another. The author describes how the mathematics of geometry, symmetry, proportion, forces, stresses and tensions arise in the construction of great buildings. He also develops several strands of basic mathematics (Euclidean and coordinate geometry in particular) from a historical perspective, showing how the mathematics relates to problems faced in the construction of these buildings.

One of the best known Greek structures is the Parthenon in Athens. It is typical of the column and beam, or post and lintel structure. This type of structure is not, however, suitable for long spans on account of the weight of the beams. The Roman solution was the development of the arch. The arch has a disadvantage, namely the outer thrust of the wedges which make up the curved top of the arch; this outward force has to be controlled by a rigid base or by a balustrade or something similar. The Roman were successful in constructing arches, singly, or in linear arrays (as in the aqueducts), or even in circular or oval arrays (as in the Colosseum). A further development was in the three-dimensional extension of the arch to a dome, a composite of arches, such as in the Pantheon in Rome. The author discusses the horizontal thrusts created by an arch, using trigonometry and force vectors, and also discusses the oval shape of the Colosseum, showing how an oval can be constructed from circular arcs.

The passion for domes, despite the problems of outward thrust, is widespread. The reader is given insights into the problems posed in the Hagia Sophia church in Byzantium, the Dome of the Rock in Jerusalem, the Cathedral in Milan, the Sancta Maria Cathedral in Florence, and St Peter's in Rome. Progress was made possible by the introduction of the Gothic arch, which is steeper than the Roman circular arc, thereby lessening the horizontal thrusts and enabling greater heights to be achieved. There is an extremely interesting comparison of four domes which look quite

similar: St Paul's Cathedral in London, Ste. Geneviève (later the Panthéon) in Paris, both eighteenth century, and the nineteenth century St Isaac's Cathedral in St Petersburg and the Capitol Building in Washington. The materials used in the structure of such domes changed over time, the latter two using cast iron.

Throughout the book the architectural story is paralleled by the development of different branches of mathematics. The development of coordinate geometry is fundamental, and during the Renaissance there is an emphasis on proportion, related to the musical intervals of Alberti. Leonardo's *Homo Vitruvianus* displays the perfect proportions of the human body, which are to be reflected in the perfect proportions of buildings. The rediscovery of the ancient Greek work on conic sections proved invaluable to scientific progress, and in art the theory of perspective involved the transformation of circles into ellipses. The final chapter of the book deals with calculus which is applied to estimate, among other things, the weight of the dome of the Hagia Sophia. It is also used to derive the hyperbolic cosine formula for the catenary curve, the locus of a hanging chain, whose inversion has been used as the shape of the Gateway Arch in St Louis. Earlier in the book there had been a discussion of weighted hanging chains, involving mathematical work of Robert Hooke who worked with Christopher Wren on St Paul's Cathedral. In St Paul's there are three shells in the dome; the outer one that is visible to all; the inner one visible from inside the cathedral, and between them a conical brick shell which supports both the outer shell and the massive lantern at the top. The conical shape of the middle one is the inversion of the shape taken by a hanging chain with weights at the bottom representing the lantern weights at the top of the shell. Hooke's principle said, 'As hangs the flexible line, so but inverted will stand the rigid arch'. Related ideas were used by the French scientist and Jesuit priest Pierre Varignon to study the stability of the dome of St Peter's, and their validity was later confirmed by work of Jacques Heyman, professor of engineering at Cambridge, in the 1960s.

Towards the end of the book, the story of Jørn Utzon's Sydney Opera House, built from 1957 to 1973, is told. As the author writes, 'its construction exhibits a very interesting tension between the demands of the creative spirit of the architect, the insistence of the engineer on staying on a buildable course, and the economic advantage of standardized production. Prestressed concrete and spherical geometry are both essential and an early version of computer analysis plays an important role'. This then leads to a final section on the impact of computers on architecture and the advances in computer-aided design and manufacture, freeing the architect from the restrictions and conventions of the past. The development of new software made Frank Gehry's design of the Bilbao Guggenheim Museum possible.

I have just skimmed over the vast wealth of information and detail provided in this fascinating and beautiful book. There are 16 pages of colour photographs, and about 330 black and white figures and photos. At the end of each chapter there is an extensive collection of mathematical problems and discussion topics, some related to the chapter's content, and some going off on tangents. Some of these are straightforward; others are much more challenging or ask for independent investigations into other buildings or wider areas of mathematics such as symmetry and Greek geometry. The book would be ideal for a school or college library, and it would grace any coffee table. Here is beautiful mathematics, related to real life applications, presented in an attractive and very readable way.

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