

# DEFENSE OF THE DOCTORAL DISSERTATION

DEPARTMENT OF MATHEMATICS

## “Coisotropy of Fixed Points Under Torus Action on the Variety of Lagrangian Subalgebras”



Song Gao

Thursday, June 27, 2024

Time: 8:00 AM

Zoom URL:

<http://notredame.zoom.us/j/98666804316>



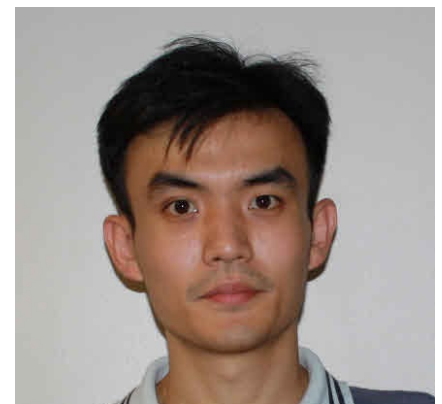
Examination Committee:

Sam Evens, Advisor

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Jiang-Hua Lu, University of Hong Kong



### Abstract:

Let  $\mathfrak{g}$  be a complex semi-simple Lie algebra with adjoint group  $G$ . Consider the standard Manin triple structure on the double Lie algebra  $\mathfrak{d} := \mathfrak{g} \oplus \mathfrak{g}$  and the associated variety of Lagrangian subalgebras  $L(\mathfrak{d})$ . In this thesis, we study the variety  $CL(\mathfrak{d})$  of coisotropic subalgebras of  $\mathfrak{g}$ , which is embedded as a subvariety of  $L(\mathfrak{d})$ . The diagonal maximal torus  $H\Delta$  in  $G \times G$  acts on  $L(\mathfrak{d})$  and  $CL(\mathfrak{d})$ . We describe the connected components of the fixed point set  $L(\mathfrak{d})^{H\Delta}$  using toric varieties, and we compute the irreducible components of the fixed point set  $CL(\mathfrak{d})^{H\Delta}$ . We provide the first examples of continuous families in  $CL(\mathfrak{d})^{H\Delta}$ . Our results may be viewed as completing and unifying the earlier works of Zambon, Kroeger, and Le.