Department of Mathematics
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LOGIC SEMINAR

Guest Speaker: Damir Dzhafarov
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Date: Thursday, April 18, 2024
Time: 2:00 PM
Location: 258 Hurley Bldg
Zoom URL: NA

Lecture Title:
The Ginsburg-Sands theorem and computability

Abstract

In their 1979 paper "Minimal Infinite Topological Spaces", Ginsburg and Sands proved that every infinite topological space has an infinite subspace homeomorphic to exactly one of the following five topologies on ω: indiscrete, discrete, initial segment, final segment, and cofinite. The proof, while nonconstructive, features an interesting application of Ramsey's theorem for pairs (RT^2_2). We analyze this principle in computability theory and reverse mathematics, using Dorais's formalization of CSC spaces. Among our results are that the Ginsburg-Sands theorem for CSC spaces is equivalent to ACA_0, while for Hausdorff spaces it is provable in RCA_0. Furthermore, if we enrich a CSC space by adding the closure operator on points, then the Ginsburg-Sands theorem turns out to be equivalent to the Chain-antichain principle (CAC). The most surprising case is that of the Ginsburg-Sands theorem restricted to T_1 spaces. Here, we show that the principle lies strictly between ACA_0 and RT^2_2, yielding perhaps the first natural theorem of ordinary mathematics (i.e., conceived outside of logic) to occupy this interval. I will discuss the proofs of both the implications and separations, which feature several novel combinatorial elements, and survey a new class of purely combinatorial principles below ACA_0 and not implied by RT^2_2 revealed by our investigation. This is joint work with Heidi Benham, Andrew DeLapo, Reed Solomon, and Java Darleen Villano.