

# DEFENSE OF THE DOCTORAL DISSERTATION

DEPARTMENT OF MATHEMATICS

## “Some results on operads and configuration spaces”



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Location: 258 Hurley Bldg.

Examination Committee:

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### Abstract:

The relation of algebraic topology and geometric topology is necessarily subtle because algebraic topology uses homotopical techniques which, if applied bluntly, lose a great deal of geometric information. One possible way to study geometry through homotopical methods is via operads. The most fundamental operads in homotopy theory, the little disks operads  $E_n$ , are constructed geometrically via moduli spaces of disks. This is not an isolated example: geometric objects often are either acted upon by operads, form operads, or have their moduli spaces form operads. One major area of study in geometric topology is the configuration spaces of nonoverlapping points in  $M$ , and more generally, embedding spaces of one manifold into another. In this thesis, we study these spaces from the perspective of operads in spectra. The category of spectra is the “linearization” of the category of spaces, and as such it has a rich algebraic structure which parallels that of the category of chain complexes. As a result, many classical techniques from the homological algebra of chain complexes can be used. One such technique is Koszul duality for operads, modules, and algebras. We initially study the stabilization of configuration space as an object in spectra, computing its Spanier–Whitehead dual and deducing the homotopy invariance of stabilized configuration space. For a framed manifold  $M$ , the collection of all configuration spaces  $\text{Conf}(M, i)$  has a right module action of the  $E_n$  operad. We study the  $E_n$  operad action on the collection of configuration spaces and deduce results about Browder brackets in the homology of configuration spaces. We also deduce homotopy invariance results concerning the associated graded of filtrations of stabilized embedding spaces. Ultimately, we prove that the stabilized configuration spaces of a framed manifold are a fixed point of Koszul duality for right modules. In other words, the collection of stabilized configuration spaces of a framed manifold is Koszul self dual. This statement generalizes Atiyah duality to include the action of the  $E_n$  operad and has applications to spaces of embeddings and configuration spaces with labels in an  $E_n$  algebra.