

HONORS ALGEBRA 3-4

Part 3. Honors Math 3. Group Theory

1. GROUPS

- Laws of Composition. Groups and Subgroups
- Cyclic Groups.
- Homomorphisms. Isomorphisms.
- Cosets. Modular Arithmetic
- The Correspondence Theorem
- Product and Quotient Groups

2. SYMMETRY

- Symmetry of Plane Figures.
- Isometries. Discrete Groups of Isometries.
- Abstract Symmetry: Group Operations. The Operation on Cosets
- The Counting Formula
- Permutation Representations
- Finite Subgroups of the Rotation Group.

3. MORE GROUP THEORY

- Cayley's Theorem
- The Class Equation
- p – Groups
- Normalizers. The Sylow Theorems. Applications
- The Free Group. Generators and Relations.

4. GROUP REPRESENTATIONS

- Definitions. Irreducible Representations.
- Unitary Representations. Characters One-Dimensional
- Characters
- The Regular Representation
- Schur's Lemma
- The Orthogonality Relations

Part 4. Honors Math 4. Rings, Fields and Galois Theory

1. RINGS

- Definition and Examples.
- Homomorphisms and Ideals. Quotient Rings.
- Product Rings
- Fractions.
- Maximal Ideals. Algebraic Geometry.

2. FACTORING

- Unique Factorization Domains.
- Gauss's Lemma
- Factoring Integer Polynomials. Gauss Primes.

3. QUADRATIC NUMBER FIELDS

- Algebraic integers. Factoring Algebraic Integers.
- Ideal Multiplication. Factoring Ideals.
- Prime Ideals and Prime Integers.
- Ideal Classes and the Class Group.

4. LINEAR ALGEBRA IN A RING

- Modules
- Free Modules. Generators and Relations.
- Noetherian Rings.
- Structure of Abelian Groups.
- Application to Linear Operators.
- Polynomial Rings in Several Variables.

5. FIELDS

- Definition and Examples.
- Field Extensions.
- Ruler and Compass Constructions.
- Finite Fields
- Function Fields.
- The Fundamental Theorem of Algebra.

6. GALOIS THEORY

- Symmetric Functions
- Splitting Fields.
- Isomorphisms of Field Extensions, Galois Extensions.
- The Main Theorem
- Cubic and Quartic Equations.