HONORS ANALYSIS 1-2

The rapid accumulation of knowledge during the 19th century lead to huge jumps in abstraction during the 20th century. The classes cover the so called modern analysis, or analysis of functions of infinitely many variables. These are not abstractions for the sake of abstraction, but they grew out of the need to address questions arising in the study of differential equations, probability and quantum physics. The ideas turned out to have applicability in many other areas of math and sciences.

Here is a more detailed description of these classes.

Part 1. Honors Analysis 1

1. Analysis on metric spaces

1.1. Metric spaces.

- Definition and examples
- Basic geometric and topological properties
- Continuity
- Connectedness
- Continuous linear maps
- Introduction to point set topology

1.2. Completness.

- Cauchy sequences
- Completions
- Applications: Banach fixed point theorem, Baire's category.

1.3. Commpactness.

- Compact metric spaces
- Compact subsets

1.4. Continuous functions on compact metric spaces.

- Arzela-Ascoli compactness theorem.
- Stone-Weierstrass approximation theorem.

2. Ordinary differential equations

2.1. Basic concepts and examples.

- The concept of differential equation
- Separable equations
- Homogeneous equations
- First order linear differential equations
- Riccati equations
- Integral inequalities

2.2. **Existence and uniqueness for the Cauchy problem.**

- Picard's existence and uniqueness theorem
- Peano's existence theorem
- Global existence and uniqueness
- Continuous dependence on initial conditions and parameteres.
- Dynamical systems

2.3. **Systems of linear differential equations.**

- General theory
- Higher order linear differential equations with constant coefficients
- The harmonic oscillator. Resonance.
- Systems of linear differential equations with constant coefficients.
- Differentiability of solutions with respect to parameters and initial conditions.
- Dynamical systems again.

Part 2. Honors Analysis 2

1. Measure theory and the modern theory of integration

1.1. Measurable spaces and measures.

- Sigma-algebras
- Measurable maps
- Measures. The “almost everywhere” terminology
- Premeasures. The Lebesgue premeasure

1.2. Construction of measures.

- The Carathéodory constructions
- The Lebesgue measure on the real line.

1.3. The Lebesgue integral.

- Definition and fundamental properties.
- Connection with the Riemann integral.
- Product measures and Fubini’s theorem.
- The Lebesgue measure in $\mathbb{R}^n$. The concept of volume.

1.4. L^p-spaces.

- Definition and Hölder’s inequality
- The Banach space L^p.
- Density results.

1.5. Signed measures.

- The Hahn and Jordan decomposition
- The Radon-Nikodym theorem.

1.6. Duality.

- The Daniell integral and the dual of $C(K)$.
- The dual of L^p.

2. Introduction to functional analysis

2.1. Hilbert spaces.

- Inner products and their associated norms.
- Orthogonal projections.
- Duality.
- Abstract Fourier decompositions

2.2. Introduction to Fourier analysis.

- Trigonometric series: the L^2-theory.
- Trigonometric series: the L^1-theory.
- Pointwise convergence of Fourier series.
- Uniqueness.

2.3. More point set topology.