HONORS MATH 1-2

The first semester of Honors Math introduces the student to the rules of mathematical logic and rigorously addresses a fundamental question: what are the numbers, and how can we operate with them. Humanity needed two millennia to elucidate these aspects. We achieve this in one semester. The remainder of the Honors Math courses builds on this foundation to rigorously reveal an important and very useful slice of mathematics known as classical real analysis. Traditional calculus is merely the cherry on top of this cake.

While addressing the "why?" questions occupies a larger fraction of time than in traditional math courses, the Honors Math courses devote a substantial amount of time explaining how these theoretical concepts work in concrete situations whose relevance transcend pure mathematics.

The classical analysis of functions of a single variable is covered during the Honors Math 1-2 classes, while the analysis of functions of several variables is discussed in the Honors Math 3-4 courses.

Below is a more detailed list of topics covered in these classes.

Part 1. Honors Math 1

1. Introduction to mathematical logic

- Logical Operations: disjunction, conjunction, negation and deduction
- Quantifiers
- Sets
- Functions

2. Foundations of mathematics

2.1. The real numbers.

- The algebraic axioms.
- The order axioms.
- The completeness axiom.

2.2. Special classes of numbers.

- The natural numbers, and the integers.
- The induction principles and applications.
- Rational and irrational numbers. Why do square roots exist?

3. The concept of limit

3.1. Sequences and series.

- Limits of sequences: definition, properties and fundamental examples.
- Bounded monotone sequences. The Euler number.
- Compactness concepts: nested intervals and Bolzano-Weierestrass theorems.
- Cauchy sequences.
- Series: convergence and absolute convergence.
- Introduction to power series.

3.2. **Limits of functions.**

- The limit of a function at a point and its basic properties.
- Rigorous construction of the elementary functions.
- Continuous functions: definition and basic properties.
- Intermediate value property, existence of extrema and uniform continuity.

Part 2. **Honors Math 2**

1. DIFFERENTIAL CALCULUS

1.1. **The derivative.**

- Linear approximations and derivatives..
- Derivatives of elementary functions.
- Derivatives of sums, products and quotients.
- The chain rule.
- The Fermat principle, Rolle and Lagrange theorem.
- Higher order derivatives.

1.2. **Application of differential calculus.**

- Taylor approximations. Minima/maxima of functions.
- Why does L'Hôpital's rule work?
- Convexity. Newton's approximation method.
- Jensen, Hölder, Minkowski, and AM-GM inequalities.
- Antiderivatives and linear first order differential equations.

2. INTEGRAL CALCULUS

- Riemann and Darboux sums. Darboux integrability theorem.
- Fundamental examples of integrable functions.
- Fundamental Theorem of Calculus.
- Integration by parts and by substitution.
- Some classical integrals and their applications: Wallis' and Stirling's formulas.
- Improper integrals.