HONORS MATH 3-4

The classes represent the higher dimensional counterparts of the Freshman classes Honors Math 1-2. However, there is a substantial jump in complexity since there are many surprising phenomena that occur in higher dimensions that are not possible in the one-dimensional case. This subject requires more sophisticated methods and new ideas. In particular, it requires a substantial and nontrivial input from linear algebra. That is why this class is taken in parallel with the Honors Algebra 1-2 classes.

Below is a more detailed list of topics covered in these classes.

Part 3. Honors Math 3

1. Euclidean geometry and topology

1.1. Basic affine geometry.

- Lines, line segments and convex subsets in $\mathbb{R}^n$.
- Affine and linear subspaces of $\mathbb{R}^n$.
- Linear operators and matrices.

1.2. Introduction to the topology of Euclidean spaces.

- The Euclidean inner product and norm. Orthogonality.
- Open sets, closed sets and convergence of sequences.
- Completeness of $\mathbb{R}^n$.
- Path connected subsets of $\mathbb{R}^n$.
- Compact subsets of $\mathbb{R}^n$: Heine-Borel and Bolzano Weierstrass theorems.
- Continuous maps. Fundamental examples and properties.

2. Differential calculus

2.1. The Frechet derivative.

- The Frechet differential of a map at a point.
- Partial derivatives and the Jacobian matrix.
- Chain rule and applications. Polar, cylindrical and spherical coordinates.
- Higher order partial derivatives.
- Div, grad, curl and the Laplacian.

2.2. Applications.

- Taylor approximations of degree at most 2. The Hessian.
- Minima and maxima of functions of several variables.
- The implicit function theorem.
- Submanifolds of $\mathbb{R}^n$. Tangent vectors and tangent spaces.
- Lagrange multipliers.

Part 4. Honors Math 4

1. Integrals of functions of several variables

1.1. Multidimensional Riemann integral.

- Darboux sums and Riemann integrability.

- Fundamental properties and examples.
- Jordan measure.
- Fubini theorem.
- Change in variables formula. Higher dimensional spherical coordinates.
- Applications.

1.2. **Improper integrals.**
- Definition and convergence criteria
- Classical improper integrals: Gamma and Beta functions, Gaussian integrals.

2. Integrals over curves and surfaces

2.1. **Integral over curves.**
- Arclength and the integral of a function over a curve.
- Differential forms of degree 1 and their integrals over oriented curves.
- Conservative vector fields and their potentials.

2.2. **Integral over surfaces in $\mathbb{R}^3$.**
- Compact surfaces (with boundary).
- The area element and the integral of a function over a surface. Area of a surface.
- The flux of a vector field through an oriented surface.
- The flux as the integral of a 2-form
- Divergence, Green and Stokes' formulas. Div, grad, curl revisited.
- Formal calculus with differential forms in $\mathbb{R}^3$. Stokes' formula revisited.