*Department of Mathematics*
*University of Notre Dame*

# *PDE, COMPLEX ANALYSIS AND DIFFERENTIAL GEOMETRY SEMINAR*

**Guest Speaker: Xiaoming Zheng**
**Central Michigan University**

***Date:*** Tuesday, November 21, 2023
***Time:*** 11:00 AM
***Location:*** 258 Hurley Bldg
***Zoom URL:*** https://notredame.zoom.us/j/98530943143

## *Lecture Title:*

## Numerical algorithms and simulations of boundary dynamic control for optimal mixing in unsteady Stokes flows

### *Abstract*

This work develops an efficient and accurate optimization algorithm to study the optimal mixing problem driven by boundary control of unsteady Stokes flows, based on the theoretical foundation laid by Weiwei Hu and Jiahong Wu in a series of work. The scalar being mixed is purely advected by the flow and the control is a force exerted tangentially on the domain boundary through the Navier slip conditions. The control design has potential applications in many industrial processes such as rotating wall driven mixing, mircomixers with acoustic waves, and artificial cilia mixing. The numerical algorithms have high complexity, high accuracy demand, and high computing expense, due to the multiscale nature of the mixing problem and the optimization requirements. A crucial problem is the computation of the Gateaux derivative of the cost functional. To this end, a hybrid approach based on variational formula and finite difference is built with high accuracy and efficiency to treat various types of control input functions. We have experimented with various optimization schemes including the steepest descent algorithm, the conjugate gradient method and two line search options (backtracking and exact line search). We are able to identify and implement the best combinations. The numerical simulations show that the mixing efficacy is limited when only one single type of control is applied, but can be enhanced when more diverse control types and more time segmentation are utilized. The mix-norm in the optimal mixing decays exponentially. The numerical study in this work demonstrates that boundary control alone could be an effective strategy for mixing in incompressible flows. This work is a collaboration with Weiwei Hu at University of Georgia and Jiahong Wu at University of Notre Dame.

Xiaoming Zheng - Central Michigan University