

## ***LOGIC SEMINAR***

**Guest Speaker: Andrew Marks**  
**University of California at Berkeley**



**Date:** Tuesday, September 19, 2023

**Time:** 2:00 PM

**Location:** 125 Hayes-Healy Hall

**Zoom URL:** NA

**Lecture Title:**

**A dichotomy characterizing piecewise Baire class alpha functions**

***Abstract***

In the 1920s, Lusin asked whether every Borel function on  $2^\omega$  is a union of countably many partial continuous functions (i.e. whether every Borel function is piecewise continuous). This question has a negative answer; an example of a non-piecewise continuous Borel function is the Turing jump. This is the only counterexample in a sense: Solecki and Zapletal have shown that every Borel function  $f$  is either piecewise continuous, or the Turing jump continuously reduces to  $f$ . We generalize the Solecki-Zapletal dichotomy throughout the Borel hierarchy. Recall that a Borel function is Baire class alpha if and only if it is  $\Sigma^{0_{\alpha+1}}$  measurable. We show that every Borel function  $f$  is either piecewise Baire class alpha, or the complete Baire class  $\alpha + 1$  function (an appropriate iterate of the Turing jump) continuously reduces to  $f$ . Our proof uses an adaptation of Montalbán's game metatheorem for priority arguments to boldface descriptive set theory.