

MATH 80430, TOPICS IN TOPOLOGY

Algebraic K-theory and the telescope conjecture

FALL 2023, MW 3:30-4:45

Hayes-Healy 125

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Topics. Algebraic K-theory has recently been revolutionized by a series of spectacular advances. Nikolaus and Scholze have dramatically simplified the construction of topological cyclic homology (TC) and TC remains one of the most effective methods of algebraic K-theory computations. Work of Hahn-Raksit-Wilson gives a new motivic filtration on TC and related spectra, giving new computational methods which allow for new direct approaches to the results of Ausoni and Rognes on $K(ku)$, while simultaneously giving a new homotopical take on the work of Bhatt-Morrow-Scholze on prismatic cohomology. Hahn-Wilson and Burklund-Schlank-Yuan have proven the Rognes red shift conjecture for K-theory, showing that algebraic K-theory raises chromatic filtration, and this summer Burklund-Hahn-Levy-Schlank used decent methods in algebraic K-theory to disprove the Telescope Conjecture, resolving what was arguably the biggest open problem in homotopy theory. I will give an overview of these results, and try to explain what they are saying and give outlines of the arguments. The class will have a homotopical focus, but I will do my best to give homotopical background depending on the composition of the audience.