

LOGIC SEMINAR

Guest Speaker: Matthew Harrison-Trainor
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Date: Tuesday, April 25, 2023

Time: 2:00 PM

Location: 125 Hayes-Healy Hall

Zoom URL: NA

Lecture Title:

The complexity of homeomorphism classes

Abstract

Given a fixed topological (for us, always Polish) space X , how hard is it to decide whether some other topological space Y is homeomorphic to X ? Measuring this tells us the complexity of classifying X up to homeomorphism. We present our topological spaces as the completions of countable metric spaces, and ask how complicated the homeomorphism class of a given space X is in the Polish space of all such metric spaces. The classical Scott analysis tells us that the analogous question for a countable structure and isomorphism is always Borel, while the metric Scott analysis says the same thing for separable metric structures and isometric isomorphism. We prove an effective version of the Gelfand duality which reduces homeomorphism of topological spaces to isomorphism of C^* -algebras. As a consequence, for a fixed X , the homeomorphism class of X is always Borel. We will also discuss examples in detail, particularly the 2-sphere (and surfaces in general), for which we show that the homeomorphism class is arithmetic.