Department of Mathematics University of Notre Dame

FELIX KLEIN SEMINAR

Speaker: Tommaso Sferruzza University of Parma

Date: Thursday, April 20, 2023 Time: 2:00 PM Location: 258 Hurley Hall Zoom URL: NA



Lecture Title:

Deformations of Special Hermitian Metrics

Abstract

The existence of a Kähler metric imposes several constraints on the topology of compact complex manifold. As a foremost example, if (M, J) is a compact complex manifold admitting a Kähler metric, i.e., a Hermitian metric g whose fundamental form $\omega(\cdot, \cdot) \coloneqq g(J \cdot, \cdot)$ is closed, then the Betti numbers $b_k \coloneqq H^k_{dR}(M; \mathbb{R})$ of M satisfy

$$\begin{cases} b_{2k} > 0, \\ b_{2k+1} \equiv 0 \mod 2. \end{cases}$$

Moreover, a compact Kähler manifold is formal according to Sullivan and the Hodge decomposition holds. Even further obstructions are given, so that, in general, a compact complex manifold does not admit such metrics. As consequence, a number of notions of special Hermitian metrics, e.g., SKT metrics, astheno-Kähler metrics, and balanced metrics, which generalize Kähler metrics and arise naturally in prominent settings of complex geometry have been recently introduced; their existence and characterization on complex manifolds are still open research topics.

In this seminar, I will present some results of my PhD thesis which relate special metrics existence problems with cohomological structures of a complex manifold. In particular, I will show that there exist obstructions in the Dolbeault cohomology and in the Bott-Chern cohomology for the existence of curves of SKT metrics (respectively, astheno-Kähler metrics, balanced metrics) along curves of deformations of the complex structure of a fixed compact complex manifold.