

# DEFENSE OF THE DOCTORAL DISSERTATION

DEPARTMENT OF MATHEMATICS

## “Determinantal Inequalities For Totally Positive Matrices”

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Tuesday, April 4, 2023

Time: 2:00 PM

Location: 210 DeBartolo Hall

Examination Committee:

Misha Gekhtman, Advisor

David Galvin

Matthew Dyer

Steven Karp



### Abstract:

Totally positive matrices are matrices in which each minor is positive. Such matrices appear in many different areas of theoretical and applied mathematics. Investigation of the structure of inequalities between the minors of totally positive matrices plays an important role in many problems in these areas. In representation theory of quantized enveloping algebras the notion of canonical bases plays an important role. One of the approaches to describe canonical bases is to study their dual objects, so called dual canonical bases. Lusztig has shown that specializations of elements of the dual canonical bases at  $q=1$  are totally non-negative polynomials. To this end there is an interest in functions that are positive on the locus of totally positive matrices. We present results on multiplicative determinantal inequalities (joint work with M. Gekhtman) as well as possible further directions including ratios containing exotic cluster variables. Furthermore, we present a majorizing monotonicity of symmetrized Fischer's products which are a natural generalization of Hadamard-Fischer inequalities. Majorizing monotonicity of symmetrized Fischer's products were already known for hermitian positive semi-definite matrices which brings additional motivation to verify if they hold for totally positive matrices as well (joint work with M. Skandera). The main tools we employed are network parametrization and Temperley-Lieb and monomial trace immanants.