Defense of the Doctoral Dissertation

Department of Mathematics

"Algebraic hyperbolicity of very general hypersurfaces and their complements"

University of Notre Dame
College of Science

Wern Yeong

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Examination Committee:
Eric Riedl, Advisor
Felix Janda
Juan Migliore
Claudiu Raicu

Abstract:

A complex algebraic variety is said to be (Brody) hyperbolic if it does not admit any non-constant holomorphic maps from $\mathbb{C}$, which are called entire curves. Conjectures by Kobayashi, Green, Griffiths, Lang, and others predict that the hyperbolicity of a variety is controlled by the positivity of its canonical bundle. More precisely, this circle of conjectures predicts that a variety of general type is hyperbolic outside of a proper subvariety called an exceptional locus, which may be empty if its canonical bundle has enough positivity. In this dissertation, we make several advances to the study of these conjectures with respect to the notion of algebraic hyperbolicity as defined by Demailly. We obtain a complete classification of very general hypersurfaces in $\mathbb{P}^m x \mathbb{P}^n$ by their bidegrees, except in the case of threefolds in $\mathbb{P}^3 x \mathbb{P}^1$. We present three techniques to do so, which build on past work by Ein, Voisin, Pacienza, Coskun, Riedl, and others. As another application of these techniques, we solve the penultimate case of degree 8 fourfolds in the widely-studied question of the algebraic hyperbolicity of very general hypersurfaces in $\mathbb{P}^n$, leaving sextic threefolds as the only open case. In a collaboration with X. Chen and E. Riedl, we develop some of the above techniques to the setting of quasi-projective varieties. We prove that the complement of a very general hypersurface in $\mathbb{P}^n$ of degree $2n$ is algebraically hyperbolic outside of a proper exceptional locus. Moreover, for the complement of a very general quartic plane curve, we show that the exceptional locus is precisely the union of the bitangent and flex lines to the curve.