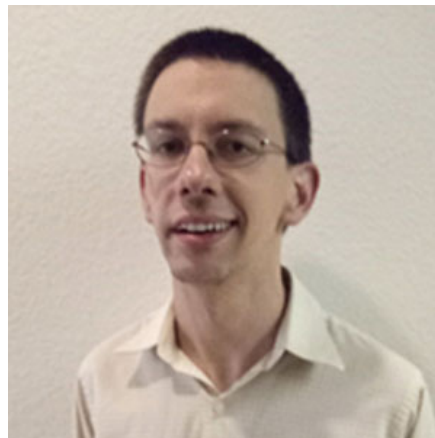


# ***ALGEBRAIC GEOMETRY AND COMMUTATIVE ALGEBRA SEMINAR***

**Speaker: Evan O'Dorney**  
**University of Notre Dame**



**Date:** Wednesday, February 1, 2023

**Time:** 3:00 PM

**Location:** 258 Hurley Hall

**Zoom URL:** NA

***Lecture Title:***

**Diophantine Approximation on Conics**

***Abstract***

Given a conic  $\mathcal{C}$  over  $\mathbb{Q}$ , it is natural to ask what real points on  $\mathcal{C}$  are most difficult to approximate by rational points of low height. For the analogous problem on the real line (for which the least approximable number is the golden ratio, by Hurwitz's theorem), the approximabilities comprise the classically studied Lagrange and Markoff spectra, but work by Cha–Kim and Cha–Chapman–Gelb–Weiss shows that the spectra of conics can vary. We provide notions of approximability, Lagrange spectrum, and Markoff spectrum valid for a general  $\mathcal{C}$  and prove that their behavior is exhausted by the special family of conics  $\mathcal{C}_n : XZ = nY^2$ , which has symmetry by the modular group  $\Gamma_0(n)$  and whose Markoff spectrum was studied in a different guise by A. Schmidt and Vulakh. The proof proceeds by using the Gross-Lucianovic bijection to relate a conic to a quaternionic subring of  $\text{Mat}^{2 \times 2}(\mathbb{Z})$  and classifying invariant lattices in its 2-dimensional representation.