Department of Mathematics
University of Notre Dame

MODEL THEORY SEMINAR

Guest Speaker: Scott Mutchnik
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Date: Tuesday, November 8, 2022
Time: 10:30 AM
Location: 125 Hayes-Healy Hall
Zoom URL: NA

Lecture Title:
Conant-independence

Abstract

The free amalgamation theories introduced by Conant (2017) axiomatize certain independence relations in homogeneous structures, such as the random graphs and the generic K_n-free graphs. Conant shows that all modular free amalgamation theories are simple or SOP_3, and this result turns out to be connected to some central open problems in the classification of unstable structures. Answering a question of Conant, we have shown that the generic constructions of Kruckman and Ramsey (2018) give examples of non-modular free amalgamation theories; we have also shown that all free amalgamation theories, even non-modular ones, are either $NSOP_1$ or SOP_3. By generalizing a version of Conant's free amalgamation axioms, we isolate two structural properties with no known $NSOP_4$ counterexamples which, together, imply that a theory is $NSOP_1$ or SOP_3. We explain how these generalized free amalgamation axioms relate these two properties, by relativizing Chernikov and Ramsey (2016) and Kaplan and Ramsey (2020)'s theory of Kim-independence in $NSOP_1$ theories to an abstract stationary independence relation outside of $NSOP_1$. When this relative version of $NSOP_1$ holds, we obtain symmetry for a recently introduced absolute independence relation, Conant-independence, which as in the strong Kim-dividing of Kaplan, Ramsey and Shelah (2019) represents forking-independence at a maximally generic scale (rather than at the "generic scale" of Kaplan and Ramsey (2020)). Symmetry for Conant-independence reveals not only the surprising significance of the class $NSOP_4$, but also new connections between the problem of extending the theory of independence beyond $NSOP_1$ and the established classification-theoretic problems of whether $NSOP_2 = NSOP_3$ and whether the higher $NSOP_n$ hierarchy is strict within NTP_2.

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