Colloquium

UNIVERSITY OF NOTRE DAME
Department of Mathematics

Speaker: Zhijun (George) Qiao
University of Texas-Pan American

Wednesday, September 28, 2011
4:15 PM
117 Hayes-Healy Hall

Title: The CH peakon system and algebro-geometric solution on a symplectic submanifold

Abstract:

The CH equation is shown to have peaked solitons (peakon) as well as the multi-peakon solutions. The CH peakon system is proven a finite-dimensional integrable system in the Louville sense and in particular, two peakons system can be explicitly solved. In the talk, I will discuss the Camassa-Holm (CH) spectral problem yielding two different integrable hierarchies of nonlinear evolution equations (NLEEs), one is of negative order CH hierachy while the other one is of positive order CH hierarchy. The two CH hierarchies possess the zero curvature representations through solving a key matrix equation. We see that the well-known CH equation is included in the negative order CH hierarchy while the Dym type equation is included in the positive order CH hierarchy. Furthermore, under two constraint conditions between the potentials and the eigenfunctions, the CH spectral problem is cast in: 1. a new Neumann-like N-dimensional system when it is restricted into a symplectic submanifold of R^{2N}, which is proven to be integrable by using the Dirac-Poisson bracket and the r-matrix process; and 2. a new Bargmann-like N-dimensional system when it is considered in the whole R^{2N}, which is proven to be integrable by using the standard Poisson bracket and the r-matrix process. The whole CH hierarchy (an integro-differential hierarchy, both positive and negative order) is shown to have the parametric solutions which obey the corresponding constraint relation. In particular, the CH equation, constrained to a symplectic submanifold in R^{2N}, and the Dym type equation have the parametric solutions. Moreover, we see that the kind of parametric solution of the CH equation is not gauge equivalent to the peakons. Solving the parametric representation of the solution on the symplectic submanifold gives a class of a new algebro-geometric solution of the CH equation.