Department of Mathematics
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Speaker: Parker Edwards

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Location: 258 Hurley Hall

Zoom URL: notredame.zoom.us/j/98589842511

Lecture Title:

A computational theory for distance functions of algebraic manifolds

Abstract

One of the main theoretical approaches in computational geometry and topology runs as follows: Let $d_X : \mathbb{R}^n \to \mathbb{R}$ be the distance-to-X function for a compact subspace $X \subseteq \mathbb{R}^n$ and let $P \subseteq \mathbb{R}^n$ be a "good" finite sample of X. For instance, one interpretation of "good" is that the ℓ_∞ distance $\|d_X - d_P\|_\infty$ is small. The goal is usually to show that an algorithm of interest correctly extracts information about d_X when using the point set P as input. Understanding the behavior of d_X and its relation to interesting properties of X, e.g., its Betti numbers, is therefore essential. The critical point theory for distance functions initiated by Grove and Shiohama in 1977 is precisely the right framework for analyzing this behavior. For most subspaces, d_X is not differentiable everywhere. With the right definition of critical points and values, however, one recovers Morse-function type behavior for d_X. In this talk, I will discuss the background and some new results for the case where X is an algebraic manifold, i.e., defined as the set of real solutions of a system of polynomial equations. Fu proved in 1985 that the set of critical values for d_X is finite when X is any compact semialgebraic set, but the proof does not translate to a feasible algorithm for computing critical values. The results are: 1. A new proof of Fu's theorem in the setting of algebraic manifolds which yields an algorithm using numerical algebraic geometry to compute critical values of d_X from the defining polynomials 2. That a large class of algebraic manifolds (generic non-quadratic complete intersections) have finitely many critical points. This is an application of the celebrated Alexander-Hirschowitz theorem. In this case we can also compute all of the critical points of d_X from defining polynomials. This is joint work with Sandra Di Rocco, David Eklund, Oliver Gäfvert, and Jonathan Hauenstein.

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