COHOMOLOGY OF LINE BUNDLES ON THE INCIDENCE CORRESPONDENCE

Abstract

by

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Given a (partial) flag variety over algebraically closed field $\mathbf{k}$, a fundamental problem is to determine the sheaf cohomology groups of line bundles on it. In characteristic zero, a complete answer is given by the Borel-Weil-Bott theorem. Over fields of positive characteristic, results such as Kempf's vanishing theorem and Anderson's characterization of (non)vanishing of H^1 provide partial answers. However, the problem still remain largely unknown in positive characteristic.

This thesis is dedicated to characterizing the (non)vanishing of cohomology of line bundles on the incidence correspondence

$$X = \{(p, H) \in \mathbb{P}V \times \mathbb{P}V^{\vee} : p \in H\},$$

where V is a vector space of dimension ≥ 3 over an algebraically closed field of characteristic $p > 0$. When $n = 3$, this is the result of Griffith from the 70s.

Our strategy is to recast the problem in terms of computing cohomology of (twisted) divided powers of cotangent sheaf on projective space. We study the vanishing behavior by estimation of Castelnuovo-Mumford regularity of composition factors induced by natural truncation of Frobenius.

We also investigate the character of cohomology groups of line bundles on the incidence correspondence with respect to the natural action of n-dimensional torus.

When $n = 3$, we give an explicit recursion formula which recovers the result of Linyuan Liu. We also provide a resursive formula in the case $n = 4, p = 2$. For the case $p = 2$, we give a conjecture of non-recursive character formula, involving symmetric polynomials naturally defined in terms of winning positions of the game of Nim. We will show the conjecture holds in the case $n = 3$ and $n = 4$.