Abstract:

Regularity is a numerical invariant that measures the complexity of the structure of homogeneous ideals in a polynomial ring. Papers of Bayer-Mumford and others give examples of families of ideals attaining doubly exponential regularity. In contrast, Bertram-Ein-Lazarsfeld, Chardin-Ulrich, and Mumford have proven that there are nice bounds on the regularity of the ideals of smooth projective varieties. As discussed in an influential paper by Bayer and Mumford (1993), the biggest missing link between the general case and the smooth case is to obtain a decent bound on the regularity of all prime ideals--the ideals that define irreducible projective varieties. The longstanding Eisenbud-Goto Regularity Conjecture (1984) predicts an elegant linear bound in terms of the degree of the variety (also called multiplicity). The conjecture was proven for curves by Gruson-Lazarsfeld-Peskine, for smooth surfaces by Lazarsfeld and Pinkham, for most smooth 3-folds by Ran, and in many other special cases. McCullough and I introduced two new techniques and used them to provide many counterexamples to the Eisenbud-Goto Regularity Conjecture. In fact, we show that the regularity of prime ideals is not bounded by any polynomial function of the degree. Starting from an arbitrary homogeneous ideal \( N \), our ideas make it possible to construct a prime ideal whose regularity, degree, (and other numerical invariants) are expressed in terms of numerical invariants of \( N \). The talk will discuss the concept of regularity and provide an overview of the current state of results on regularity of prime ideals.