Department of Mathematics
University of Notre Dame

MODEL THEORY SEMINAR

Guest Speaker: Benjamin Castle
University of Notre Dame

Date: Tuesday, February 8, 2022
Time: 10:30 AM
Location: 125 Hayes-Healy Hall
Zoom URL: NA

Lecture Title:
Restricted Trichotomy in Characteristic Zero, I

Abstract

Zilber's Restricted Trichotomy Conjecture predicts that the Zilber trichotomy should hold for strongly minimal structures interpreted in algebraically closed fields. The general strategy for solving this problem -- which involves finding an abstract definition of `tangency' -- dates back to the 1980s. However the strategy only goes through smoothly if a sufficient fragment of the Zariski topology can be definably reconstructed inside the interpreted structure -- and this task has proven to be quite difficult in full generality. In the most successful approach to date on the problem, a preprint of Hasson and Sustretov gives a full proof whenever the universe of the interpreted structure is a curve -- but instead of trying to recover the topology directly, their proof works by finding a way to run the argument without `encountering' certain unwanted topological phenomena (namely the isolated points of certain sets). Unfortunately the techniques used by Hasson and Sustretov are specific to curves, and the case of higher dimensional constructible sets has remained wide open. In these talks, I will present a recent solution of the full conjecture for the field of complex numbers, thus also for any field of characteristic zero. More precisely, I will discuss a new result which detects certain closure points of definable sets using purely combinatorial data -- thus giving exactly the topological fragment needed for the conjecture to go through. This `closure result' generalizes a technique of Peterzil and Starchenko used in classifying o-minimally definable expansions of the complex field; most notably, while Peterzil and Starchenko's technique has been used before for `low dimensional' sets, the new version is able to handle sets of arbitrary dimension. Once this result has been discussed, the full proof of Zilber's conjecture will be surprisingly accessible. Precisely, at this point we will obtain (1) a relatively straightforward reduction to the case of curves, and (2) a simplified version of Hasson and Sustretov's argument in the curve case.