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ALGEBRAIC GEOMETRY AND COMMUTATIVE ALGEBRA SEMINAR

Speaker: Andres Fernandez Herrero
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Date: Tuesday, October 5, 2021
Time: 2:30 PM
Location: 125 Hayes-Healy Hall
Zoom URL:

Lecture Title:
Moduli of sheaves via affine Grassmannians

Abstract

A useful tool in the study of the moduli space of stable vector bundles on a smooth curve C is the existence of the Mumford compactification, which is constructed by adding a boundary parametrizing semistable vector bundles. If the smooth curve C is replaced by a higher dimensional variety X, then one can compactify the moduli problem by allowing vector bundles to degenerate to an object known as a "torsion-free sheaf". Gieseker and Maruyama constructed moduli spaces of semistable torsion-free sheaves on such a variety X. More generally, Simpson proved the existence of moduli spaces of semistable pure sheaves supported on smaller subvarieties of X. All of these constructions use the methods of geometric invariant theory (GIT). The moduli problem of sheaves on X is more naturally parametrized by a geometric object M called an "algebraic stack". In this talk I will explain an alternative GIT-free construction of the moduli space of semistable pure sheaves that is intrinsic to the moduli stack M. This approach also yields a Harder-Narasimhan stratification of the unstable locus of the stack. Our main technical tools are the theory of Θ-stability introduced by Halpern-Leistner and some recent methods developed by Alper, Halpern-Leistner and Heinloth. In order to apply these recent results, one needs to show some monotonicity conditions for a polynomial numerical invariant on the stack. We prove monotonicity by defining a higher dimensional analogue of the affine Grassmannian for pure sheaves. If time allows, I will also explain how these ideas can be applied to some other moduli problems. This talk is based on joint work with Daniel Halpern-Leistner and Trevor Jones.