Lecture Title: Lech's Inequality for the Buchsbaum-Rim Multiplicity and Mixed Multiplicities

Abstract
The Hilbert-Samuel multiplicity of m-primary ideals is an important invariant of Noetherian local rings. In some sense it measures the singularities of these rings. Singularities are detected by seeing how much the multiplicity differs from the colength. For instance, a ring is Cohen-Macaulay if the colength of a parameter ideal equals its multiplicity. One is then interested in bounding the multiplicity of an ideal, possibly by its colength. Lech proved such a bound. He originally proved this result to establish his conjecture in low dimensions. The inequality itself is of interest however, and it is far from sharp. A recent improvement to Lech's bound is due to Huneke, Smirnov, and Validashti. They improved it by giving an analogous bound for the multiplicity of a given ideal times the maximal ideal. In this talk I'll give a brief discussion of multiplicities, along with its generalizations. I'll then talk about Lech type bounds for the different multiplicities. This is joint work with Kelsey Walters.