MATH 80520 Spring 2021. A. Pillay. MWF 12.45 – 2.00 pm, room 208, DeBartolo Hall, beginning February 3rd.

CONTINUOUS LOGIC.

This is a slight extension of usual first order (finitary) logic, where formulas are real-valued (or have values in a compact space) rather than just having values true, false.

Nevertheless, the compactness theorem is still valid, and (families of) compact spaces play a big role.

Continuous logic goes back to Chang and Keisler in the 1960's. Ward Henson developed a version (``approximate logic") in the 70's and 80's, aimed at formalizing a Los theorem for ``metric" ultraproducts. Among the ideas was that such a logic should relevant to the objects of functional analysis in the same way that classical first order logic is relevant to algebraic structures.

This was combined with Ben-Yaacov's compact abstract theories to obtain in the early 2000' s, a certain popular formalism which is sometimes meant when saying "continuous logic".

Some versions were anticipated by Shelah (and Krivine). Hrushovski's "Robinson theories", the generalization to the category of existentially closed models (Pillay), and the development of hyperdefinability and hyperimaginaries within the classical first order context, are also closely related.

Anyway in the course we will try to give a self contained treatment of continuous logic, taking into account some of the formalisms above, as well as discussing applications.

We will mention references to papers and books in the course itself. But let us mention now one of the seminal papers (possibly rather formal), "Model theory of metric structures" by Ben Yaacov, Berenstein, Henson, Usvyatsov, in *Model Theory with Applications to Algebra and Analysis, Vol. II*, eds. Z. Chatzidakis, D. Macpherson, A. Pillay, and A.Wilkie, Lecture Notes series of the London Mathematical Society, No. 350, Cambridge University Press, 2008, 315--427. Also available on Henson's web-page: https://faculty.math.illinois.edu/~henson/