Department of Mathematics
University of Notre Dame

Patrick Heslin - University of Notre Dame

GRADUATE STUDENT GEOMETRY SEMINAR

Guest Speaker: Patrick Heslin

University of Notre Dame

Date: Friday, November 6, 2020

Time: 3:00 PM

Location: Zoom

Zoom URL: notredame.zoom.us/j/96302373523?pwd=SWZMOTFuRCt2UEIrdGk0bStjc0M2Zz09

Lecture Title:

A regularity property of the L^2 exponential map on the space of Volumorphisms

Abstract

In this talk we are concerned with a certain regularity property of exponential maps on the space of Sobolev s-regular volumorphisms of the 2-torus. **Defintion:** An exponential map on $D^s_\mu(\mathrm{T}^2)$ is said to exhibit *smoothing* if given initial data $u_0 \in T_e D^s_\mu(\mathrm{T}^2)$:

$$exp_e(u_0) \in D^{s+1}_\mu(\mathbb{T}^2) \Rightarrow u_0 \in T_e D^{s+1}_\mu(\mathbb{T}^2)$$

Kappeler, Loubet and Topalov observed this property in their study of the full group of (orientation preserving) diffeomorphisms of the 2-torus, equipped with various right-invariant H^r metrics. Their goal was to show that, under certain restrictions, the arising exponential maps were C^1 diffeomorphisms in the sense of Frechet. The earliest result of this nature however, to the best of the author's knowledge, is due to Constantin and Kolev where they used this smoothing property to show the existence of a C^1-Frechét exponential map on C^∞ diffeomorphisms of the circle, equipped with a right-invariant H^r metric, for $r \in \mathbb{Z}_+$, covering the case of the Camassa-Holm equation. Most notably, in two dimensions, we cover the case of the L^2 exponential map, where the reduction of the geodesic equation to the Lie Algebra of H^s vector fields yields the Euler equations.