University of Notre Dame Department of Mathematics

PDE, COMPLEX ANALYSIS AND DIFFERENTIAL GEOMETRY SEMINAR

Adriano Tomassini

University of Parma

Will give a lecture entitled:

On cohomological decomposability of almost-complex structures

On

Tuesday, May 10, 2011

At

11:00 AM

In

258 Hurley Hall

Abstract

Recently, some authors have studied cohomological properties of compact almost-complex manifolds (see e.g. [3], [4], [5] and the references therein).
In this talk we will focus on $\mathcal{C}^\infty$*-pure-and-full almost-complex structures.* More precisely, let J be an almost-complex structure on a $2n$-dimensional manifold M. Then J acts in a natural way on the bundle of 2-forms $\Lambda^2(M)$, and

$$\Omega^2(M) = \Omega^{1,1}_J(M)_{\mathbb{R}} \oplus \left(\Omega^{2,0}_J(M) + \Omega^{0,2}_J(M)\right)_{\mathbb{R}},$$

where $\Omega^2(M)$ denotes the space of sections of $\Lambda^2(M)$ and $\Omega^{1,1}_J(M)_{\mathbb{R}}$, $\left(\Omega^{2,0}_J(M) + \Omega^{0,2}_J(M)\right)_{\mathbb{R}}$ are the J-invariant and J-anti-invariant 2-forms respectively.
Therefore, it is natural to consider the subgroups $H^{1,1}_J(M)_{\mathbb{R}}$, $H^{(2,0),(0,2)}_J(M)_{\mathbb{R}}$ of $H^2_{dR}(M;\mathbb{R})$ whose elements are cohomological classes which admit a representative of pure type $(1,1)$, $(2,0)+(0,2)$ respectively.
In the terminology of T.-J.- Li and W. Zhang (see [3]), an almost-complex structure J on M is said to be $\mathcal{C}^\infty$*-pure-and-full* if

$$H^2_{dR}(M;R) = H^{1,1}_J(M)_{\mathbb{R}} \oplus H^{(2,0),(0,2)}_J(M)_{\mathbb{R}}.$$

In [4], T. Draghici, T.-J. Li and W. Zhang have proved that

every almost-complex strutcure on a 4-dimensional compact manifold is $\mathcal{C}^\infty$*-pure-and-full.*

This result can be viewed as a Hodge decomposition for compact 4-dimensional almost-complex manifolds.