**Lecture Title:** Geometric applications of the Laplace equation on Ricci-flat manifolds

**Abstract**
I will discuss the Laplace equation on a complete Ricci-flat manifold with Euclidean volume growth. In the case when a tangent cone at infinity of the manifold has smooth cross section, the Green function for the Laplace equation can be used to define a functional which measures the distance between a certain scale of the manifold and the cross section of the tangent cone. Using the Łojasiewicz inequality of Colding-Minicozzi for this functional, we describe how two arbitrarily far apart scales in the manifold can be identified in a natural way. I will also discuss a matrix Harnack inequality for the Green function when there is an additional condition on sectional curvature, which is an analogue of various matrix Harnack inequalities obtained by Hamilton and Li-Cao in time-dependent settings.