Abstract
An important part of an intro algebraic topology course is to define what a manifold is and figure out what being a manifold gets you. The most important thing it gets you is Poincaré duality, a relation between the homology and cohomology of the manifold. Being a manifold is far from homotopy invariant, but having Poincaré duality is. In this talk we find necessary and sufficient conditions that a finite CW complex is homotopy equivalent to a compact, simply connected manifold of dimension $2k+1>4$.