

Department of Mathematics
University of Notre Dame

Logic Seminar

Speaker: Chieu-Minh Tran
University of Notre Dame

Title: *Structures of sets with minimal measure growth
in connected unimodular groups*

Date: Tuesday, June 16, 2020

Location: Zoom: <https://notredame.zoom.us/j/763507156>

Time: 11:00 am

Abstract:

Let G be a connected unimodular group equipped with a (left and hence right) Haar measure μ_G , and suppose $A, B \subseteq G$ are nonempty and compact. An inequality by Kemperman gives us

$$\mu_G(AB) \geq \min\{\mu_G(A) + \mu_G(B), \mu_G(G)\},$$

implying that the n -fold product A^n of A has $\mu_G(A^n) \geq \min\{n\mu_G(A), \mu_G(G)\}$ and when G is not compact, $\lim_{n \rightarrow \infty} \mu_G(A^n)/(n\mu_G(A)) \geq 1$. We obtain simple characterizations of G , A , and B such that the equalities hold. This is the first general result of its kind in nonabelian continuous settings and, at the same time, provides a complete answer to a question asked by Kemperman in 1964. We also get near equality versions of the above results with uniform linear bound for connected compact groups, confirming conjectures made by Griesmer and by Tao. For the special case of Lie groups, we develop new techniques to construct homomorphisms into tori. This can be applied to obtain the first general measure expansion gap result for connected compact simple Lie groups. The reduction from locally compact groups to Lie groups uses the Gleason–Yamabe Theorem, and the uniform error bound uses ideas from Hrushovski’s Lie model theorem and Breuillard–Green–Tao’s classification of approximate groups. (Joint with Yifan Jing)