

University of Notre Dame Department of Mathematics

FELIX KLEIN SEMINAR

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Will give a lecture entitled:

Critical sets of random linear combinations of eigenfunctions.

On

Thursday, February 3, 2011

At

2:00 PM

In

125 Hayes-Healy Hall

Abstract

Given a compact, m -dimensional Riemann manifold (M, g) and a large positive constant L we denote by U_L the subspace of $C^\infty(M)$ spanned by the eigenfunctions of the Laplacian corresponding to eigenvalues $\leq L$. We equip U_L with the standard Gaussian probability measure induced by the L^2 -metric on U_L , and we denote by N_L the expected number of critical points of a random function in U_L . We prove that $N_L \sim C_m \dim U_L$ as $L \rightarrow \infty$, where C_m is an explicit positive constant that depends only on the dimension m and satisfying the asymptotic estimate $\log C_m \sim \frac{m}{2} \log m$ as $m \rightarrow \infty$. The proof uses simple probabilistic ideas to reduce the estimates of N_L to estimates of the spectral function of the Laplacian, and the estimates of C_m to Wigner's semi-circle theorem in random matrix theory.