

# Colloquium

University of Notre Dame  
Department of Mathematics

Zhouli Xu - Massachusetts Institute of Technology

**Speaker:** Zhouli Xu

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**Will give a lecture entitled**

The geography problem on 4-manifolds:  $10/8 + 4$

**Date:** Monday, February 3, 2020

**Time:** 4:00 PM

**Location:** 229 Hayes-Healy Hall

**Departmental Tea:** Tea in Room 257 (lounge in Hurley Hall) at 3:30 p.m.

**Abstract:**

A fundamental problem in 4-dimensional topology is the following geography question: "which simply connected topological 4-manifolds admit a smooth structure?" After the celebrated work of Kirby-Siebenmann, Freedman, and Donaldson, the last uncharted territory of this geography question is the "11/8-Conjecture". This conjecture, proposed by Matsumoto, states that for any smooth spin 4-manifold, the ratio of its second-Betti number and signature is least  $11/8$ . Furuta proved the " $10/8+2$ "-Theorem by studying the existence of certain  $\text{Pin}(2)$ -equivariant stable maps between representation spheres. In this talk, we will present a complete solution to this problem by analyzing the  $\text{Pin}(2)$ -equivariant Mahowald invariants. In particular, we improve Furuta's result into a " $10/8+4$ "-Theorem. Furthermore, we show that within the current existing framework, this is the limit. This is joint work with Mike Hopkins, Jianfeng Lin and XiaoLin Danny Shi.