Department of Mathematics
University of Notre Dame

LOGIC SEMINAR

Guest Speaker: C. Ward Henson
University of Illinois at Urbana-Champaign

Date: Tuesday, November 5, 2019
Time: 2:00 PM
Location: 125 Hayes-Healy Hall

Lecture Title:
Model theory of R-trees and of ultrametric spaces

Abstract

First, we consider the class of metric spaces (M, d) that are R-trees with a convex metric. To treat this class using continuous first order logic, we fix a base point p in M and require that M have radius at most r with respect to $p (r > 0)$. The class of these structures (M, d, p) is axiomatizable. Moreover, the theory of this class has a model companion T, whose models we describe precisely. This theory is a well behaved continuous theory. For example, T has QE and is complete; it is stable (but not superstable) and has the maximum possible number of models in each infinite cardinal. Second, given a model $M = (M, d, p)$ of T, we consider the closed subset $E_r(M) := x in M | d(p, x) = r$. This is a definable set for T, and the entire structure M can be reconstructed from $(E_r(M), d)$. The metric d on $E_r(M)$ is an ultrametric; further, at every x in $E_r(M)$, the set of distances $d(x, y) | y$ in $E_r(M)$ is dense in the interval $[0, 2r]$. These properties are easily seen to be axiomatizable in continuous logic, and we let T^* denote the resulting theory. We show that T^* has QE, so it is complete; further, T and T^* are bi-interpretable. This is joint work with Sylvia Carlisle.