

University of Notre Dame Department of Mathematics

# ALGEBRAIC GEOMETRY AND COMMUTATIVE ALGEBRA SEMINAR

**Bernadette Boyle & Megan Patnott**

University of Notre Dame

*Will give two lectures entitled:*

**The unimodality of pure O-sequences of type three in three variables  
and  
Minimal Free Resolutions of General Points on Cubic Surfaces**

Tuesday, October 26, 2010

*At*

2:00PM

*In*

125 Hayes-Healy Hall

## ***Abstracts***

Bernadette Boyle

"The unimodality of pure O-sequences of type three in three variables"

In this presentation we will look at some properties of the Hilbert functions of monomial algebras, particularly to see when they are unimodal. Due to Macaulay's theorem, one knows that algebras in two variables are unimodal. Furthermore, it has been shown that monomial Artinian level algebras of type two in three variables have the Weak Lefschetz Property, and thus are unimodal. On the other hand, for any  $r > 2$ , there exists a monomial Artinian level algebra in  $r$  variables whose Hilbert function fails unimodality with an arbitrary number of peaks. In this presentation, we will show the unimodality of the Hilbert function in the smallest open case, namely that of monomial Artinian level algebras of type three in three variables. Since the Weak Lefschetz Property does not necessarily hold for such algebras, we give a new approach.

Megan Patnott

"Minimal Free Resolutions of General Points on Cubic Surfaces"

The Minimal Resolution Conjecture (MRC), roughly stated, says that the graded minimal free resolution of a general set of points in  $P^n$  has no "ghost terms." A generalization of the MRC for arbitrary varieties in  $P^n$ , given by Mustata, predicts that the graded Betti numbers of the ideal of such a variety completely determine those of the ideal of a general set of points on it. In particular, it predicts that the graded minimal free resolution of a general set of points on an arbitrary variety,  $X$ , in  $P^n$  has no ghost terms, except where forced to do so by  $X$ . Casanellas showed that this generalized MRC holds for  $t$  general points on a smooth cubic surface in  $P^3$  for certain special values of  $t$ , using Gorenstein liaison. Our result extends hers by verifying the conjecture for all  $t$  and allowing the cubic surface to have isolated double points. We give an overview of the work done on both the MRC and the generalized MRC, and then discuss our result.

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# GRADUATE STUDENT SEMINAR

**Matthew Dyer and Fred Xavier**

University of Notre Dame

*Will give two lectures entitled:*

**“Coxeter groups, root systems and Bruhat orders”**

**&**

**“Good shadows, localization at infinity, maximum principles and convex hulls.”**

Friday, December 10, 2010

***At***

4:30PM

***In***

231 Hayes-Healy Hall

## ***Abstracts***

Matthew Dyer (4:30PM)

**“Coxeter groups, root systems and Bruhat orders”**

Coxeter groups are (possibly infinite) reflection groups which generalize the finite groups generated by orthogonal reflections in real Euclidean spaces. They include dihedral groups and finite symmetric groups, and also the finite and affine Weyl groups which are important in Lie theory. I'll discuss some results and open questions involving a few of the many natural algebraic and combinatorial structures which are associated to Coxeter groups.

Fred Xavier (5:00PM)

**“Good shadows, localization at infinity, maximum principles and convex hulls.”**

Any non-empty open convex subset  $C$  in  $n$ -space is the convex hull of some complete non-compact submanifold  $M$ , of any pre-assigned codimension. On the other hand, there are obstructions for  $C$  to be such a convex hull if the geometry of  $M$  is, a priori, suitably controlled at infinity by means of its Grassmannian-valued Gauss map. We will show how this question can be approached using our recent conceptual refinement of the celebrated Omori-Yau asymptotic maximum principle in geometric analysis. We also introduce a new approach to these maximum principles, based on ideas from dynamics. This is joint work with F. Fontenele.

There will be pizza following the lectures provided by the department.