

## ***TOPOLOGY SEMINAR***

**Guest Speaker: Peter Haine**  
**MIT**



**Date:** Tuesday, September 10, 2019

**Time:** 2:30 PM

**Location:** 258 Hurley Hall

**Lecture Title:**

**Stratified spaces, constructible sheaves, & exit-paths**

***Abstract***

A natural question arises when working with intersection cohomology and other stratified invariants of singular manifolds: what is the correct stable homotopy theory for these invariants to live in? But before answering that question, one first has to identify the correct unstable homotopy theory of stratified spaces. The exit-path construction of MacPherson, Treumann, and Lurie provides functor from suitably nice stratified topological spaces to “abstract stratified homotopy types” —  $\infty$ -categories with a conservative functor to a poset. Moreover, just like how functors out of the fundamental  $\infty$ -groupoid classify locally constant sheaves of spaces, functors out of the exit-path  $\infty$ -category classify constructible sheaves of spaces. Work of Ayala–Francis–Rozenblyum even shows that their conically smooth stratified topological spaces embed into the  $\infty$ -category of abstract stratified homotopy types. In the first part of the talk, we’ll explain the basics of stratified spaces and the exit-path construction, as well as how to go further and produce an equivalence between the homotopy theory of all stratified topological spaces and these “abstract stratified homotopy types”. In the second part of the talk, we’ll discuss joint work with Barwick and Glasman where we construct an version of the exit-path category in the algebra-geometric setting the classifies constructible sheaves in the étale topology. This “étale exit-path  $\infty$ -category” is a powerful invariant of schemes, and is even a complete invariant of normal varieties over a number field. It also provides a new and simple description of the étale homotopy type of Artin–Mazur–Friedlander. If time permits, we’ll survey some new avenues coming from this line of thinking, as well as how to interpret some old results of Friedlander in étale homotopy theory from this perspective.