



**Speaker:** Paul Shafer  
Cornell University

Thursday, November 4, 2011  
1:00 PM  
117 Hayes-Healy Hall

**Title:** Medvedev degrees: characterizing the first-order theory and the join-irreducibles

**Abstract:**

Medvedev introduced his degrees to formalize Kolmogorov's idea of a "calculus of problems" and a "logic of problem solving." A subset  $X$  of Baire space represents a mathematical problem, and  $X$  Medvedev reduces to  $Y$  if there is a uniform effective procedure (i.e., a Turing functional) which produces elements of  $X$  from elements of  $Y$ . This notion of reducibility captures the idea of one problem being at least as hard as another. We show that this ordering of problems is as complicated as possible by showing that the first-order theory of the Medvedev degrees is recursively isomorphic to the third-order theory of arithmetic. We also characterize the join-irreducible Medvedev degrees as the degrees of the complements of Turing ideals and discuss implications for modeling propositional logics by Medvedev degrees.