

University of Notre Dame Department of Mathematics

# ALGEBRAIC GEOMETRY AND COMMUTATIVE ALGEBRA SEMINAR

**Bernadette Boyle & Megan Patnott**

University of Notre Dame

*Will give two lectures entitled:*

**The unimodality of pure O-sequences of type three in three variables  
and  
Minimal Free Resolutions of General Points on Cubic Surfaces**

Tuesday, October 26, 2010

*At*

2:00PM

*In*

125 Hayes-Healy Hall

## **Abstracts**

Bernadette Boyle

"The unimodality of pure O-sequences of type three in three variables"

In this presentation we will look at some properties of the Hilbert functions of monomial algebras, particularly to see when they are unimodal. Due to Macaulay's theorem, one knows that algebras in two variables are unimodal. Furthermore, it has been shown that monomial Artinian level algebras of type two in three variables have the Weak Lefschetz Property, and thus are unimodal. On the other hand, for any  $r > 2$ , there exists a monomial Artinian level algebra in  $r$  variables whose Hilbert function fails unimodality with an arbitrary number of peaks. In this presentation, we will show the unimodality of the Hilbert function in the smallest open case, namely that of monomial Artinian level algebras of type three in three variables. Since the Weak Lefschetz Property does not necessarily hold for such algebras, we give a new approach.

Megan Patnott

"Minimal Free Resolutions of General Points on Cubic Surfaces"

The Minimal Resolution Conjecture (MRC), roughly stated, says that the graded minimal free resolution of a general set of points in  $P^n$  has no "ghost terms." A generalization of the MRC for arbitrary varieties in  $P^n$ , given by Mustata, predicts that the graded Betti numbers of the ideal of such a variety completely determine those of the ideal of a general set of points on it. In particular, it predicts that the graded minimal free resolution of a general set of points on an arbitrary variety,  $X$ , in  $P^n$  has no ghost terms, except where forced to do so by  $X$ . Casanellas showed that this generalized MRC holds for  $t$  general points on a smooth cubic surface in  $P^3$  for certain special values of  $t$ , using Gorenstein liaison. Our result extends hers by verifying the conjecture for all  $t$  and allowing the cubic surface to have isolated double points. We give an overview of the work done on both the MRC and the generalized MRC, and then discuss our result.