

**Speaker:** Cameron Hill  
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Friday, October 15, 2010  
3:00 pm  
126 DeBartolo Hall

**Title:** Geometric model theory in efficient computability

**Abstract:**

This talk, in two parts, will consist of a sketch of the proof of a single main result linking geometric ideas from the first-order model theory of infinite structures with complexity-theoretic analyses of problems over classes of finite structures. To remove any suspense, the statement of the theorem is as follows:

**Theorem 1.** *Let  $K = \text{fin}[T^G]$ , where  $T$  is a complete  $k$ -variable theory with infinitely many finite models up to isomorphism.*

*I. If  $T$  is constructible, then  $K$  is rosy.*

*II.  $T$  is efficiently constructible if and only if  $K$  is super-rosy.*

Obviously, a great number of definitions are needed (regardless of the readers background, most likely) to make sense of these assertions. For the time being, it should be understood as a shadow of the “main current” of first-order model theory – namely, Shelah’s Classification theory. I take “efficiently constructible” – meaning that models of  $T$  can be efficiently recovered from elementary diagrams of subsets – to be a reasonable substitute for “classifiable” in the classical theory. We then seek a hierarchy of structural properties culminating in efficient constructibility in analogy with the stability-theoretic hierarchy,  $\text{Stable} \supseteq \text{Super-stable} \supseteq \text{Classifiable} = \text{Super-stable} + \text{NDOP}$ . In the classical scenario, any non-trivial bound on the number of models of the theory in each cardinality imposes stability, which already supports the rudimentary notion of geometry known as non-forking independence. In the scenario of this study, the hypothesis of constructibility by an algorithm cursorily *imitating* that of an efficient algorithm in form (meaning, an essentially inflationary program which isn’t necessarily efficient) is sufficient to impose another rudimentary notion of geometry on the class of models – in this case, known as  $\mathfrak{p}$ -independence in a rosy class;<sup>1</sup> this is the content of *I* of the theorem. The further requirement of efficiency – polynomially-bounded running times – induces a further guarantee of good behavior in the geometry of  $\mathfrak{p}$ -independence, and the “only if” portion of *II* of the theorem amounts to just this fact. It turns out, then, that this additional tractability in the geometry gives enough purchase to devise an efficient algorithm, initially disguised as a weak model-theoretic coordinatization result, for the class of the theory’s finite models.

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<sup>1</sup>The character  $\mathfrak{p}$  is pronounced “thorn.”

In the first hour, I will present the rudiments of  $\mathfrak{b}$ -independence in and super/rosiness of Fraïssé classes like  $\text{fin}[T^G]$ , and having done that, I will sketch the coordinatization theorem which makes model-building possible through iterating the algebraic closure operator. In the second hour, I will present the object-creating relational model of computation in more detail. With this in hand, I will explain the construction of the unfolding digraph of a program evaluation and the layered construction of a model-theoretic independence relation from d-separation in unfolding digraphs.