Logic Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

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Tuesday, October 12, 2010
3:45 pm
125 Hayes-Healy Hall

Title: Geometric model theory in efficient computability

Abstract:

This talk, in two parts, will consist of a sketch of the proof of a single main result linking geometric ideas from the first-order model theory of infinite structures with complexity-theoretic analyses of problems over classes of finite structures. To remove any suspense, the statement of the theorem is as follows:

Theorem 1. *Let* $K = fin[T^G]$*, where* T *is a complete* k*-variable theory with infinitely many finite models up to isomorphism.*

I. If T *is constructible, then* K *is rosy.*

II. T *is efficiently constructible if and only if* K *is super-rosy.*

Obviously, a great number of definitions are needed (regardless of the readers background, most likely) to make sense of these assertions. For the time being, it should be understood as a shadow of the "main current" of first-order model theory – namely, Shelah's Classification theory. I take "efficiently constructible" – meaning that models of T can be efficiently recovered from elementary diagrams of subsets – to be a reasonable substitute for "classifiable" in the classical theory. We then seek a hierarchy of structural properties culminating in efficient constructibility in analogy with the stability-theoretic hierarchy, Stable$\supsetneq$Super-stable$\supsetneq$Classifiable=Super-stable+NDOP. In the classical scenario, any non-trivial bound on the number of models of the theory in each cardinality imposes stability, which already supports the rudimentary notion of geometry known as non-forking independence. In the scenario of this study, the hypothesis of constructibility by an algorithm cursorily *imitating* that of an efficient algorithm in form (meaning, an essentially inflationary program which isn't necessarily efficient) is sufficient to impose another rudimentary notion of geometry on the class of models – in this case, known as þ-independence in a rosy class;[1] this is the content of I of the theorem. The further requirement of efficiency – polynomially-bounded running times – induces a further guarantee of good behavior in the geometry of þ-independence, and the "only if" portion of II of the theorem amounts to just this fact. It turns out, then, that this additional tractability in the geometry gives enough purchase to devise an efficient algorithm, initially disguised as a weak model-theoretic coordinatization result, for the class of the theory's finite models.

[1]The character þ is pronounced "thorn."

In the first hour, I will present the rudiments of þ-independence in and super/rosiness of Fraïssé classes like $\mathrm{fin}[T^G]$, and having done that, I will sketch the coordinatization theorem which makes model-building possible through iterating the algebraic closure operator. In the second hour, I will present the object-creating relational model of computation in more detail. With this in hand, I will explain the construction of the unfolding digraph of a program evaluation and the layered construction of a model-theoretic independence relation from d-separation in unfolding digraphs.