## Department of Mathematics University of Notre Dame

## CLUSTER ALGEBRAS SEMINAR

Speaker: Li Yu

University of Chicago

Date: Thursday, January 31, 2019

**Time:** 3:00 PM

Location: Math Bunker, Hayes-

## Lecture Title:

Quantum boson algebra and Poisson geometry of the flag variety

## Abstract

Fix a root system  $\Phi$ . In this talk we will construct Kashiwara operators associated to each positive root in  $\Phi$  acting on the negative half of the quantum group associated to  $\Phi$ . These Kashiwara operators allow us to define a variant of Kashiwaras quantum boson algebra. We study PBW, Levendorskii-Soibelman and Leibniz type properties of the boson algebra, both at the quantum and the quasi-classical levels. In particular, we will explain in detail the Poisson geometry of the quasi-classical limit P of the positive half of the quantum boson algebra. As an algebra, P turns out to be the polynomial algebra generated by the quasi-classical limit of the Kashiwara operators. We prove that P has a structure of a symmetric Poisson CGL extension, hence a structure of a cluster algebra by the work of Goodearl and Yakimov. In fact, we will establish a Poisson isomorphism from P to the coordinate ring of the open Bruhat cell in the flag variety of type  $\Phi$  equipped with the standard Poisson structure. In type  $A_n$ , we construct vector fields  $F_{i,j}$  ( $1 \le i \le j \le n$ ) such that the Schouten bracket of  $F_{i,j}$  with the Poisson bivector  $\pi$  on Spec(P) is a Poisson bivector which is compatible with  $\pi$ .