Topology Seminar

UNIVERSITY OF NOTRE DAME
Department of Mathematics

Speaker: Robert Kotiuga
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Tuesday, December 4, 2018
2:30 PM
258 Hurley Hall

Title: Eigenvalue problems for the curl operator, 3D geometric inverse problems, ...and 7D analogies

Abstract:

This talk will focus on how the curl EVP differs from the more familiar curlcurl EVP, its unique features for tackling inverse problems involving force-free magnetic fields, and how a 7D analogy helps formalize unintuitive aspects.

Articulating why topological aspects of 3D finite element (FE) meshes could be deceptively unintuitive, while 2D was so obvious, was a challenge decades ago. In FE for electromagnetics, homology calculations can be reduced to sparse matrix linear algebra enabling certain complexities of 3D geometries to be automated. The quadratic constraint imposed by the Lorentz force is a game changer which forced the embrace of 3D topological complexities. In a previous paper the topological aspects of near force-free magnetic fields have been tied to the Giroux correspondence. Taming this increased complexity, the curl EVP has emerged as a very important bridge between topological characterizations of optimal designs involving the Lorentz force, and computational linear algebra. To elaborate on the curl EVP it is important to contrast it with the curlcurl EVP. The curlcurl EVP has a direct interpretation in terms of Hodge theory. In particular, the prescription of boundary conditions is a central issue.

3D manifolds with boundary are unique in that three is the highest dimension where the middle dimensions relate to the generators and relations of presentations of the fundamental group. The use of the curl EVP is not only essential in the context of the force-free magnetic fields, but also for understanding Reidemeister torsion in the context of the Ray-Singer-Muller-Cheerger Theorem, Ed Witten's TQFT approach to the Jones polynomial, and in analytic number theory through the work of Venkatesh and Lipnowski.

Finally, we appeal to the 4k-1 point of view, with k=1,2, in order to establish a connection between open book decompositions in the context of Michele Kervaire's higher dimensional knot theory, and a generalization of the force-free magnetic field problem.