The Grassmannian is a smooth moduli space with very rich geometry that parameterizes simple varieties, namely linear spaces. One can study a "natural" generalization, the component of a Hilbert scheme that parameterizes a pair of linear spaces in $\mathbb{P}^n$. In this talk we will describe a rigidity result that allows us to completely control degenerations in this component. We will then use it to give new examples of smooth components and describe them as blowups of certain (products of) Grassmannians. Time permitting, we will also describe the singularities of other components meeting these smooth components.